EE4490 Hour Exam 1, Spring 2018

Each problem is worth 10 points. All units are mks and are considered part of the answer. Show your work for full credit.

1. The CO$_2$ molecule has an asymmetric stretch mode at 0.292 eV and a symmetric stretch mode at 0.172 eV. If $B = \frac{\hbar}{8\pi^2 c^2 l} = 39 m^{-1}$, draw an energy-level diagram and find the wavelength of the transition from J=1 of the asymmetric vibrational mode to J=0 of the symmetric vibrational mode.

\[
\begin{align*}
E_{J=1}^{\text{as}} &= J(J+1)\frac{\hbar}{c} = 2.8 h\nu \\
&= 2 \left(3.9 \text{ eV}^{-1}\right) 4.236 \times 10^{-15} \text{ ev} = 3 \times 10^{-5} \text{ eV} \\
&= 2 \left(4.49 \times 10^{-5} \text{ eV}\right) = 9.48 \times 10^{-5} \text{ eV} \\
\Delta E &= 9.68 \times 10^{-5} \text{ eV} + (0.192 - 0.172) \\
&= 0.1201 \text{ eV} = \frac{\hbar c}{\lambda} \\
\lambda &= 10.33 \text{ mm}
\end{align*}
\]

2. Determine the emission frequency width necessary to have a temporal coherence of 20m at a source wavelength of 632nm.

\[
\lambda = 632.8 \text{ nm} \\
\Delta \lambda = 20 \Rightarrow \frac{\lambda^2}{\Delta \lambda} = \frac{C}{\Delta \nu} \\
\Delta \nu = \frac{C}{\Delta \lambda} = \frac{3 \times 10^8 \text{ m/s}}{20 \text{ nm}} = 1.5 \times 10^7 \text{ Hz}
\]
3. How many photons per second are emitted from a 5mW He-Ne laser, with wavelength 632.8nm?

\[
5 \times 10^{-3} \text{[J/s]} = n \frac{hc}{\lambda} = n \frac{h \nu}{\lambda}
\]

\[
\therefore n = \frac{5 \times 10^{-3} \text{[J/s]}}{c \times 6 \times 10^{-34} \text{[J/s]} \times 3 \times 10^{15} \text{[1/s]}} = 1.6 \times 10^{16} \text{[photon/s]}
\]

4. A helium-neon laser with a gain length of \(L = 0.25\) m is installed in a two-mirror cavity having mirror reflectivities of 99% and 98%. Assuming scattering losses of \(a_1 = a_2 = 0.001\) and absorption loss \(\alpha = 0.01\), find the threshold gain, \(g_{th}\).

\[
g_{th} = \frac{1}{2L} \ln \left[ \frac{R_1 R_2 (1-a_1) (1-a_2)}{R_1 (1-a_2)(1-a_3)} \right] + \alpha
\]

\[
= 0.0906 \text{[m}^{-1}]
\]