EE4441 Hour Exam 1, Spring 2015

Each problem is worth 6 points. All units are mks and are considered part of the answer. Show your work for full credit.

1. Two electromagnetic waves of nearly equal wavelength (589 nm) are measured on a spectrum analyzer to have a frequency difference of $2 \times 10^8$ Hz. While traveling through a dispersive medium, an interferometer is used to measure their wavelength difference as 0.001nm. What is the group velocity for this wave combination in that medium?

$$\lambda = 589 \text{nm}, \quad \Delta \nu = 2 \times 10^8 \text{Hz}, \quad \Delta \lambda = 0.001 \text{nm}$$

$$v_g = \frac{\Delta \nu}{\Delta k} = \frac{2\pi \Delta \nu}{\Delta k}, \quad \Delta k = \frac{2\pi}{\lambda}, \quad \Delta \lambda$$

$$\therefore \quad v_g = \lambda^2 \frac{\Delta \nu}{\Delta \lambda} = 6.94 \times 10^{-3} \text{Hz}$$

2. Determine the emission frequency width necessary to have a temporal coherence of 20m at a source wavelength of 632nm.

$$\lambda = 632 \text{nm}, \quad \theta_c = 20 = \frac{\lambda^2}{\Delta \lambda}$$

$$\Delta \lambda = \frac{\lambda^2}{\theta_c} = 2 \times 10^{-4} \text{m}$$

$$\Delta \nu = \frac{\nu}{\lambda^2} \Delta \lambda = \frac{c}{\lambda^2} \frac{\lambda}{\theta_c} = \frac{c}{\theta_c} = \frac{3 \times 10^8 \text{Hz}}{20 \text{nm}}$$

$$= 1.5 \times 10^7 \text{Hz}$$
3. Suppose you measure a transition wavelength of 72.9 nm for an \( n=3 \) to \( n=2 \) transition. If you know that the atom is hydrogen-like (1 electron orbiting a nucleus of \( Z \) protons), what is the nuclear charge, \( Z \), of this atom?

\[
\nu = \frac{\lambda}{\lambda} = \frac{Z^2}{\lambda} \frac{E_0}{n^2} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)
\]

\[
= \frac{3 \times 10^8 \text{[Hz]}}{72.9 \text{[nm]}} = \frac{Z^2}{\lambda} \left( 3.29 \times 10^5 \text{[Hz]} \right) \left( \frac{1}{4} - \frac{1}{9} \right)
\]

\[
Z^2 = 9
\]

\[
\therefore Z = 3 \quad (L_2^{2+})
\]

4. What temperature must a He-Ne laser be at if the measured Doppler width is \( 2 \times 10^9 \text{ Hz} \)? Assume a single isotope of Ne\(^{20}\).

\[
\Delta \nu_D = 2 \times 10^9 \text{[Hz]} = 7.2 \times 10^{-5} \nu_0 \sqrt{\frac{I}{M_N}}
\]

\[
I = \left( \frac{2 \times 10^9}{7.2 \times 10^3} \frac{6.626 \times 10^{-34}}{3 \times 10^8 \text{[m/s]}} \right)^2 20
\]

\[
= 687 K
\]
5. Estimate the rotational energy level for J=1 in diatomic nitrogen, N₂, if the nitrogen bond length is \(2 \times 10^{-10} m\) and the atomic weight of nitrogen is 14 atomic mass units (and one atomic mass unit = 940MeV/c²=1.67 \times 10^{-27} kg).

\[
E_J = \frac{L^2}{2I} = \frac{J(J+1) \hbar^2}{2I}
\]

\[I = \sum m_i r_i^2 = 2(14)940\text{MeV}/c^2 (10^{-10} m)^2 = 3 \times 10^{-23} \text{[eV}\cdot\text{s}^2]\]

\[E^1 = \frac{2 \left(6.6\times 10^{-16} \text{[eV}\cdot\text{s}^2]\right)^2}{2 \left(3 \times 10^{-23} \text{[eV}\cdot\text{s}^2]\right)} = 1.2 \times 10^{-4} \text{[eV]}\]

3