\[ y = b \]
\[ v = 0 \]

\[ \nabla^2 V = 0 \]

\[ X(x) = B_1 \cosh A_1 x + B_2 \sinh A_1 x \]

\[ Y(y) = B_3 \cos A_1 y + B_4 \sin A_1 y \]

\[ A_1 = \frac{m \pi}{b} \]
\[ \frac{1}{X} \frac{d^2X}{d x^2} + \frac{1}{Y} \frac{d^2Y}{d y^2} = 0 \]

\[ V = X(x) Y(y) \]

\[ V = \sum A_1^2 + A_2^2 = 0 \]

\[ V_0 = \delta_{mn} \]

\[ V(x, y) = \sum_{m=1}^{\infty} 4V_0 \frac{\sinh \left( \frac{m \pi x}{b} \right) \sin \left( \frac{m \pi a}{b} \right)}{m \pi \sinh \left( \frac{m \pi b}{b} \right)} \]
"Frequency" domain (spectral) modeling.

Reasons for expanding functions in terms of other functions:

- Original function is intractable, yet can be expanded in terms of functions which are tractable (e.g., analytic integration).
A series expansion may naturally grow out of a PDE separated in some coordinate system.

Diagram:

```
0-------------------0
|                    |
|                    |
|                    |
|                    |
0-------------------0
```

Last problem
Common series: power series

- power of $x$ are easy integrate, differentiate, etc.

- usually easy to obtain (Taylor series at a point).

- but, successive powers of $x$ are not orthogonal, derivatives get out of hand for large $x^n$
and they do not approximate periodic functions very well.

Fourier series:
- terms are orthogonal
- derivatives nearly bounded
- expresses periodic functions very well
But both Fourier and power series do not handle functions with either vertical or horizontal asymptotes.

Role of symmetry:
- can often simplify an expression.

Role of orthogonality
- associated with efficiency.
Consider:

\[ f(\theta) = a_1 g_1(\theta) + \ldots + a_K g_K(\theta) \]

Suppose we decide to use the first \( K \) members of \( \{g_n(\theta)\} \) and expand as well as we can using the criterion of minimizing the least-squared integrated residual.
That is, choose $\alpha_i$ so as to minimize:

$$S = \int_{-\pi}^{\pi} \left[ f(\theta) - a_i g_i(\theta) \ldots - a_k g_k(\theta) \right]^2 \, d\theta$$

Examining $\alpha_i$ for the moment,

$$\frac{\partial S}{\partial \alpha_i} = 0 = 2 \int_{-\pi}^{\pi} \left[ f(\theta) - a_i g_i(\theta) \ldots - a_k g_k(\theta) \right] g_i(\theta) \, d\theta$$