PH2400 Exam 2, Fall 2017

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1. Find the kinetic energy (in terms of Planck’s constant) of a baseball (mass=1kg) confined to a one-dimensional box that is 25cm wide if the baseball can be treated as a wave in the ground state. (5 points)

\[ \text{k.e.} = \frac{p^2}{2m} \quad \text{and} \quad p = \frac{\hbar k}{i} \quad \text{so} \quad k = \frac{\hbar \pi}{L} \]

\[ \therefore \quad \text{k.e.} = \frac{\hbar^2 \pi^2}{2 (1kg) L^2} = 8 \frac{\hbar^2 \pi^2}{L^2} = 2 \hbar^2 \]

2. An electron is in the 5D_{3/2} state. What are the possible values for the z component of the electron’s total angular momentum \( J_z \)? (4 points)

\[ j = \frac{3}{2} \quad \Rightarrow \quad J_z = m_j \hbar \]

\[ m_j = j, j - 1, \ldots -j \]

\[ = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \]

\[ \therefore \quad J_z = \frac{3}{2} \hbar, \frac{1}{2} \hbar, -\frac{1}{2} \hbar, -\frac{3}{2} \hbar \]
3. Three identical non-interacting particles of mass \( m \) are placed into a 2-dimensional box with dimensions \( L_x = L \) and \( L_y = L/2 \). Find the lowest energy of the system in terms of \( L, m, \) and constants if:

(a) the particles obey the Pauli exclusion principle (3 points)

\[
E = \frac{\hbar^2 \pi^2}{2mL_x^2} \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right) = \frac{\hbar^2 \pi^2}{2mL^2} \left( n_x^2 + \gamma n_y^2 \right)
\]

\[ \therefore \text{Two particles in } n_x = n_y = 1 \text{ state} \]

\[ \text{One particle in } n_x = 2, n_y = 1 \text{ state} \]

\[ E = \frac{\hbar^2 \pi^2 L^2}{2mL_x^2} \left[ \frac{(2)(1+4) + (1)(4+4)}{18} \right] = \frac{\hbar^2 \pi^2}{mL_x^2} \left( \frac{9}{2} \right) \]

(b) the particles do not obey the Pauli exclusion principle (3 points)

\[ \forall \text{ all } 3 \text{ particles in } n_x = n_y = 1 \text{ state} \]

\[ E = \frac{\hbar^2 \pi^2}{2mL_x^2} \left( 1 + \gamma \right) = \frac{\hbar^2 \pi^2}{mL_x^2} \left( \frac{15}{2} \right) \]
4. Given that a particle has a properly-normalized wavefunction: \( \psi(x) = e^{-|x|} \) over \(-\infty \leq x \leq \infty\), find the probability that the particle is found between \(0 \leq x \leq 1\). (5 points)

\[
\text{Probability} = \int_{0}^{1} \psi^* \psi \, dx = \int_{0}^{1} e^{-2x} \, dx = \frac{(1 - e^{-2})}{2} \\
= 0.43 \\
(\text{or } 43\%) 
\]