PH2400 Exam 2, Spring 2005

Each problem is worth 4 points. Show your work for full credit.

Useful formulae:

\[ \hbar = 1.05 \times 10^{-34} \text{J} \cdot \text{s} \]
\[ \int \sin^2(ax) \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \]
\[ \int \cos^2(ax) \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \]

1. Find the kinetic energy (in terms of Planck’s constant) of a baseball (mass=1kg) confined to a one-dimensional box that is 25cm wide if the baseball can be treated as a wave in the ground state.

\[ K.E. = \frac{p^2}{2m} \]
\[ p = \hbar \cdot k \]
\[ k = \frac{n\pi}{L} \]

So

\[ K.E. = \frac{\hbar^2 \pi^2}{2(1\text{e})^2(1\text{e})^2} = 2 \cdot h^2 = 87.8 \times 10^{-68} \text{J} \cdot \text{s} \]

2. A particle is in the first excited state of a two-dimensional box of length 1 m on a side. What is its momentum (in kg m/s)?

\[ k^2 = (n_x^2 + n_y^2) \frac{\pi^2}{L^2} \]
\[ 1 = \text{excited state} \quad n_x = 2, \quad n_y = 1 \]

\[ p = \hbar \cdot k = \frac{\hbar \pi \sqrt{5}}{L} = 1.05 \times 10^{-34} \pi \sqrt{5} \]
\[ = 7.38 \times 10^{-27} \text{kg} \cdot \text{m/s} \]
3. The wavefunction for a particle confined to a one-dimensional box of length L is given by \( \Psi(x) = A \sin(n\pi x/L) + B \cos(n\pi x/L) \). Find the constants, A and B.

\[
\Psi(0) = 0 \Rightarrow B = 0
\]

\[
\int_0^L \psi^* \psi \, dx = 1 \Rightarrow \frac{A^2}{L} \left[ \frac{L}{L} - \frac{\cos \left( \frac{n\pi x}{L} \right)}{\cos \left( \frac{n\pi x}{L} \right)} \right] = 1
\]

\[
A^2 \left[ \frac{L}{L} \right] = 1
\]

\[
A = \sqrt{\frac{2}{L}}
\]

4. Find the allowed values of \( l \) for a \( Li^{2+} \) ion in the \( n=3 \) shell.

\( Li^{2+} \) is 1s, 2s, 3s, \( l \) like

\( n = 3 \Rightarrow l = 0, 1, 2 \)
5. A $Li^{2+}$ ion undergoes a transition from the $n=4$ to the $n=3$ state. What is the energy of the emitted photon (in eV)?

\[ E_n = -\frac{13.6 Z^2}{n^2} \Rightarrow E_{4\rightarrow3} = 13.6 \left( \frac{1}{\frac{4}{9}} - \frac{1}{\frac{16}{9}} \right) \]

\[ = 5.95 \text{ eV} \]

6. An electron is in the $4D_{3/2}$ state. What are the possible values for the $z$ component of the electron’s total angular momentum?

\[ \tilde{J} = \frac{3}{2} \Rightarrow J_z = m_j \hbar \]

\[ m_j = \tilde{J}, \tilde{J} - 1, \ldots, -\tilde{J} \]

\[ = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \]

\[ J_z = \frac{3}{2} \hbar, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \frac{5}{2}, -\frac{3}{2} \]