PH2400 Quiz 6, Spring 2015

Show your work for full credit.

1. A particle of mass $m$ is in a three-dimensional box with sides $L_x = L, L_y = L, L_z = L/2$.

   (a) What is the energy of the first-excited state (in terms of $m, L$, and constants)?

   $$E = \frac{\pi^2 \hbar^2}{2m} \left( \frac{n_x^2}{L^2} + \frac{n_y^2}{L^2} + \frac{n_z^2}{\left(\frac{L}{2}\right)^2} \right) = \frac{\pi^2 \hbar^2}{2m} \left( n_x^2 + n_y^2 + 4n_z^2 \right)$$

   $n_x n_y n_z = \frac{n_x^2 + n_y^2 + 4n_z^2}{6}$; 1st excited state is when $n_x = 1, n_y = 2, n_z = 1$

   \(\frac{1}{2}, \frac{2}{1}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\)  \(\text{or, } n_x = 2, n_y = 1, n_z = 1\)

   $$E_{\text{1st excited}} = \frac{\pi^2 \hbar^2}{2mL^2} (9)$$

   (b) What is the degeneracy of the first-excited state??

   Two ways to achieve 1st excited state energy

   Therefore degeneracy = 2

2. Consider singly-ionized (hydrogen-like) helium.

   (a) What are the states which have energy lower than the 3s state?

   \[3s, \ 1s, 2s, 2p\] states are below 3s

   \[2s, 2p\]

   \[1s\]

   (b) What is the degeneracy of the 4d state, neglecting spin?

   For a "d" state, \(l = 2\)

   degeneracy = \(2l + 1 = 5\)