At the end of the 19th century, scientists believed that they had learned most of what there was to know about physics. Newton’s laws of motion and his universal theory of gravitation, Maxwell’s theoretical work in unifying electricity and magnetism, and the laws of thermodynamics and kinetic theory employed mathematical methods to successfully explain a wide variety of phenomena.

However, at the turn of the 20th century, a major revolution shook the world of physics. In 1900 Planck provided the basic ideas that led to the quantum theory, and in 1905 Einstein formulated his special theory of relativity. The excitement of the times is captured in Einstein’s own words: “It was a marvelous time to be alive.” Both ideas were to have a profound effect on our understanding of nature. Within a few decades, these theories inspired new developments and theories in the fields of atomic, nuclear, and condensed-matter physics.

Although modern physics has led to a multitude of important technological achievements, the story is still incomplete. Discoveries will continue to be made during our lifetime, many of which will deepen or refine our understanding of nature and the world around us. It is still a “marvelous time to be alive.”
1.1 SPECIAL RELATIVITY

Light waves and other forms of electromagnetic radiation travel through free space at the speed \( c = 3.00 \times 10^8 \) m/s. As we shall see in this chapter, the speed of light sets an upper limit for the speeds of particles, waves, and the transmission of information.

Most of our everyday experiences deal with objects that move at speeds much less than that of light. Newtonian mechanics and early ideas on space and time were formulated to describe the motion of such objects, and this formalism is very successful in describing a wide range of phenomena. Although Newtonian mechanics works very well at low speeds, it fails when applied to particles whose speeds approach that of light. Experimentally, one can test the predictions of Newtonian theory at high speeds by accelerating an electron through a large electric potential difference. For example, it is possible to accelerate an electron to a speed of 0.99c by using a potential difference of several million volts. According to Newtonian mechanics, if the potential difference (as well as the corresponding energy) is increased by a factor of 4, then the speed of the electron should be doubled to 1.98c. However, experiments show that the speed of the electron—as well as the speeds of all other particles in the universe—always remains less than the speed of light, regardless of the size of the accelerating voltage. In part because it places no upper limit on the speed that a particle can attain, Newtonian mechanics is contrary to modern experimental results and is therefore clearly a limited theory.

In 1905, at the age of 26, Albert Einstein published his special theory of relativity. Regarding the theory, Einstein wrote,

The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties, using only a few very convincing assumptions. . . .

Although Einstein made many important contributions to science, the theory of relativity alone represents one of the greatest intellectual achievements of the 20th century. With this theory, one can correctly predict experimental observations over the range of speeds from rest to speeds approaching the speed of light. Newtonian mechanics, which was accepted for over 200 years, is in fact a limiting case of Einstein’s special theory of relativity. This chapter and the next give an introduction to the special theory of relativity, which deals with the analysis of physical events from coordinate systems moving with constant speed in straight lines with respect to one another. Chapter 2 also includes a short introduction to general relativity, which describes physical events from coordinate systems undergoing general or accelerated motion with respect to each other.

In this chapter we show that the special theory of relativity follows from two basic postulates:

1. The laws of physics are the same in all reference systems that move uniformly with respect to one another. That is, basic laws such as

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\[ \sum F = \frac{dp}{dt} \] have the same mathematical form for all observers moving at constant velocity with respect to one another.

2. The speed of light in vacuum is always measured to be \(3 \times 10^8\) m/s, and the measured value is independent of the motion of the observer or of the motion of the source of light. That is, the speed of light is the same for all observers moving at constant velocities.

Although it is well known that relativity plays an essential role in theoretical physics, it also has practical applications, for example, in the design of particle accelerators, global positioning system (GPS) units, and high-voltage TV displays. Note that these devices simply will not work if designed according to Newtonian mechanics! We shall have occasion to use the outcomes of relativity in many subsequent topics in this text.

### 1.2 THE PRINCIPLE OF RELATIVITY

To describe a physical event, it is necessary to establish a frame of reference, such as one that is fixed in the laboratory. Recall from your studies in mechanics that Newton’s laws are valid in inertial frames of reference. An inertial frame is one in which an object subjected to no forces moves in a straight line at constant speed—thus the name “inertial frame” because an object observed from such a frame obeys Newton’s first law, the law of inertia. Furthermore, any frame or system moving with constant velocity with respect to an inertial system must also be an inertial system. Thus there is no single, preferred inertial frame for applying Newton’s laws.

According to the principle of Newtonian relativity, the laws of mechanics must be the same in all inertial frames of reference. For example, if you perform an experiment while at rest in a laboratory, and an observer in a passing truck moving with constant velocity performs the same experiment, Newton’s laws may be applied to both sets of observations. Specifically, in the laboratory or in the truck a ball thrown up rises and returns to the thrower’s hand. Moreover, both events are measured to take the same time in the truck or in the laboratory, and Newton’s second law may be used in both frames to compute this time. Although these experiments look different to different observers (see Fig. 1.1, in which the Earth observer sees a different path for the ball) and the observers measure different values of position and velocity for the ball at the same times, both observers agree on the validity of Newton’s laws and principles such as conservation of energy and conservation of momentum. This implies that no experiment involving mechanics can detect any essential difference between the two inertial frames. The only thing that can be detected is the relative motion of one frame with respect to the other. That is, the notion of absolute motion through space is meaningless, as is the notion of a single, preferred reference frame. Indeed, one of the firm philosophical principles of modern science is that all observers are equivalent and that the laws of nature must take the same mathematical form for all observers. Laws of physics that exhibit the same mathematical form for observers with different motions at different locations are said to be covariant. Later in this section we will give specific examples of covariant physical laws.

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2An example of a noninertial frame is a frame that accelerates in a straight line or rotates with respect to an inertial frame.
Figure 1.1 The observer in the truck sees the ball move in a vertical path when thrown upward. (b) The Earth observer views the path of the ball as a parabola.

Figure 1.2 An event occurs at a point $P$. The event is observed by two observers in inertial frames $S$ and $S'$, in which $S'$ moves with a velocity $v$ relative to $S$.

In order to show the underlying equivalence of measurements made in different reference frames and hence the equivalence of different frames for doing physics, we need a mathematical formula that systematically relates measurements made in one reference frame to those in another. Such a relation is called a transformation, and the one satisfying Newtonian relativity is the so-called **Galilean transformation**, which owes its origin to Galileo. It can be derived as follows.

Consider two inertial systems or frames $S$ and $S'$, as in Figure 1.2. The frame $S'$ moves with a constant velocity $v$ along the $xx'$ axes, where $v$ is measured relative to the frame $S$. Clocks in $S$ and $S'$ are synchronized, and the origins of $S$ and $S'$ coincide at $t = t' = 0$. We assume that a point event, a physical phenomenon such as a lightbulb flash, occurs at the point $P$. An observer in the system $S$ would describe the event with space–time coordinates $(x, y, z, t)$, whereas an observer in $S'$ would use $(x', y', z', t')$ to describe the same event. As we can see from Figure 1.2, these coordinates are related by the equations

\[
\begin{align*}
    x' &= x - vt \\
    y' &= y \\
    z' &= z \\
    t' &= t
\end{align*}
\]

These equations constitute what is known as a **Galilean transformation of coordinates**. Note that the fourth coordinate, time, is assumed to be the same in both inertial frames. That is, in classical mechanics, all clocks run at the same rate regardless of their velocity, so that the time at which an event occurs for an observer in $S$ is the same as the time for the same event in $S'$. Consequently, the time interval between two successive events should be the same.
for both observers. Although this assumption may seem obvious, it turns out to be incorrect when treating situations in which \( v \) is comparable to the speed of light. In fact, this point represents one of the most profound differences between Newtonian concepts and the ideas contained in Einstein’s theory of relativity.

Exercise 1  Show that although observers in \( S \) and \( S' \) measure different coordinates for the ends of a stick at rest in \( S \), they agree on the length of the stick. Assume the stick has end coordinates \( x = a \) and \( x = a + l \) in \( S \) and use the Galilean transformation.

An immediate and important consequence of the invariance of the distance between two points under the Galilean transformation is the invariance of force. For example if \( F = \frac{kqQ}{(x_2 - x_1)^2} \) gives the electric force between two charges \( q, Q \) located at \( x_1 \) and \( x_2 \) on the \( x \)-axis in frame \( S \), \( F' \), the force measured in \( S' \), is given by \( F' = \frac{kqQ}{(x_2' - x_1')^2} = F \) since \( x_2' - x_1' = x_2 - x_1 \). In fact any force would be invariant under the Galilean transformation as long as it involved only the relative positions of interacting particles.

Now suppose two events are separated by a distance \( dx \) and a time interval \( dt \) as measured by an observer in \( S \). It follows from Equation 1.1 that the corresponding displacement \( dx' \) measured by an observer in \( S' \) is given by \( dx' = dx - v dt \), where \( dx \) is the displacement measured by an observer in \( S \). Because \( dt = dt' \), we find that

\[
\frac{dx'}{dt'} = \frac{dx}{dt} - v
\]

or

\[
u_x' = u_x - v
\]

where \( u_x \) and \( u_x' \) are the instantaneous velocities of the object relative to \( S \) and \( S' \), respectively. This result, which is called the Galilean addition law for velocities (or Galilean velocity transformation), is used in everyday observations and is consistent with our intuitive notions of time and space.

To obtain the relation between the accelerations measured by observers in \( S \) and \( S' \), we take a derivative of Equation 1.2 with respect to time and use the results that \( dt = dt' \) and \( v \) is constant:

\[
\frac{dx'}{dt'} = a_x' = a_x
\]

Thus observers in different inertial frames measure the same acceleration for an accelerating object. The mathematical terminology is to say that lengths (\( dx \)), time intervals, and accelerations are \textit{invariant} under a Galilean transformation. Example 1.1 points up the distinction between invariant and covariant and shows that \textit{transformation equations, in addition to converting measurements made in one inertial frame to those in another, may be used to show the covariance of physical laws.}
Example 1.1 $F_x = ma_x$ Is Covariant Under a Galilean Transformation

Assume that Newton’s law $F_x = ma_x$ has been shown to hold by an observer in an inertial frame $S$. Show that Newton’s law also holds for an observer in $S'$ or is covariant under the Galilean transformation, that is, has the form $F'_x = m'a'_{x'}$. Note that inertial mass is an invariant quantity in Newtonian dynamics.

Solution Starting with the established law $F_x = ma_x$, we use the Galilean transformation $a'_x = a_x$ and the fact that $m' = m$ to obtain $F'_x = m'a'_{x'}$. If we now assume that $F_x$ depends only on the relative positions of $m$ and the particles interacting with $m$, that is, $F_x = f(x_2 - x_1, x_3 - x_4, \ldots)$, then $F'_x = F_x$, because the $\Delta x$’s are invariant quantities. Thus we find $F'_x = m'a'_{x'}$ and establish the covariance of Newton’s second law in this simple case.

Exercise 2 Conservation of Linear Momentum Is Covariant Under the Galilean Transformation. Assume that two masses $m_1$ and $m_2$ are moving in the positive $x$ direction with velocities $v_1$ and $v_2$ as measured by an observer in $S’$ before a collision. After the collision, the two masses stick together and move with a velocity $v'$ in $S'$. Show that if an observer in $S'$ finds momentum to be conserved, so does an observer in $S$.

The Speed of Light

It is natural to ask whether the concept of Newtonian relativity and the Galilean addition law for velocities in mechanics also apply to electricity, magnetism, and optics. Recall that Maxwell in the 1860s showed that the speed of light in free space was given by $c = (\mu_0 \varepsilon_0)^{-1/2} = 3.00 \times 10^8$ m/s. Physicists of the late 1800s were certain that light waves (like familiar sound and water waves) required a definite medium in which to move, called the ether,⁴ and that the speed of light was $c$ only with respect to the ether or a frame fixed in the ether called the ether frame. In any other frame moving at speed $v$ relative to the ether frame, the Galilean addition law was expected to hold. Thus, the speed of light in this other frame was expected to be $c - v$ for light traveling in the same direction as the frame, $c + v$ for light traveling opposite to the frame, and in between these two values for light moving in an arbitrary direction with respect to the moving frame.

Because the existence of the ether and a preferred ether frame would show that light was similar to other classical waves (in requiring a medium), considerable importance was attached to establishing the existence of the special ether frame. Because the speed of light is enormous, experiments involving light traveling in media moving at then attainable laboratory speeds had not been capable of detecting small changes of the size of $c \pm v$ prior to the late 1800s. Scientists of the period, realizing that the Earth moved rapidly around

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⁴It was proposed by Maxwell that light and other electromagnetic waves were waves in a luminiferous ether, which was present everywhere, even in empty space. In addition to an overbroad name, the ether had contradictory properties since it had to have great rigidity to support the high speed of light waves yet had to be tenuous enough to allow planets and other massive objects to pass freely through it, without resistance, as observed.
the Sun at 30 km/s, shrewdly decided to use the Earth itself as the moving frame in an attempt to improve their chances of detecting these small changes in light velocity.

From our point of view of observers fixed on Earth, we may say that we are stationary and that the special ether frame moves past us with speed \( v \). Determining the speed of light under these circumstances is just like determining the speed of an aircraft in a moving air current or wind, and consequently we speak of an "ether wind" blowing through our apparatus fixed to the Earth. If \( v \) is the velocity of the ether relative to the Earth, then the speed of light should have its maximum value, \( c + v \), when propagating downwind, as shown in Figure 1.3a. Likewise, the speed of light should have its minimum value, \( c - v \), when propagating upwind, as in Figure 1.3b, and an intermediate value, \( (c^2 - v^2)^{1/2} \), in the direction perpendicular to the ether wind, as in Figure 1.3c. If the Sun is assumed to be at rest in the ether, then the velocity of the ether wind would be equal to the orbital velocity of the Earth around the Sun, which has a magnitude of about \( 3 \times 10^4 \) m/s compared to \( c = 3 \times 10^8 \) m/s. Thus, the change in the speed of light would be about 1 part in \( 10^4 \) for measurements in the upwind or downwind directions, and changes of this size should be detectable. However, as we show in the next section, all attempts to detect such changes and establish the existence of the ether proved futile!

1.3 THE MICHELSON–MORLEY EXPERIMENT

The famous experiment designed to detect small changes in the speed of light with motion of an observer through the ether was performed in 1887 by American physicist Albert A. Michelson (1852–1931) and the American chemist Edward W. Morley (1838–1923).\(^4\) We should state at the outset that the outcome of the experiment was negative, thus contradicting the ether hypothesis. The highly accurate experimental tool perfected by these pioneers to measure small changes in light speed was the Michelson interferometer, shown in Figure 1.4. One of the arms of the interferometer was aligned along the direction of the motion of the Earth through the ether. The Earth moving through the ether would be equivalent to the ether flowing past the Earth in the opposite direction with speed \( v \), as shown in Figure 1.4. This ether wind blowing in the opposite direction should cause the speed of light measured in the Earth’s frame of reference to be \( c - v \) as it approaches the mirror \( M_2 \) in Figure 1.4 and \( c + v \) after reflection. The speed \( v \) is the speed of the Earth through space, and hence the speed of the ether wind, and \( c \) is the speed of light in the ether frame. The two beams of light reflected from \( M_1 \) and \( M_2 \) would recombine, and an interference pattern consisting of alternating dark and bright bands, or fringes, would be formed.

During the experiment, the interference pattern was observed while the interferometer was rotated through an angle of \( 90^\circ \). This rotation would change the speed of the ether wind along the direction of the arms of the interferometer. The effect of this rotation should have been to cause the fringe pattern to shift slightly but measurably. Measurements failed to show any change in the

interference pattern! The Michelson–Morley experiment was repeated by other researchers under various conditions and at different times of the year when the ether wind was expected to have changed direction and magnitude, but the results were always the same: *No fringe shift of the magnitude required was ever observed.*

The negative results of the Michelson–Morley experiment not only meant that the speed of light does not depend on the direction of light propagation but also contradicted the ether hypothesis. The negative results also meant that it was impossible to measure the absolute velocity of the Earth with respect to the ether frame. As we shall see in the next section, Einstein's postulates compactly explain these and a host of other perplexing questions, relegating the idea of the ether to the ash heap of history. Light is now understood to be a phenomenon that requires no medium for its propagation. As a result, the idea of an ether in which these waves could travel became unnecessary.

**Details of the Michelson–Morley Experiment**

To understand the outcome of the Michelson–Morley experiment, let us assume that the interferometer shown in Figure 1.4 has two arms of equal length $L$. First consider the beam traveling parallel to the direction of the ether wind, which is taken to be horizontal in Figure 1.4. According to Newtonian mechanics, as the beam moves to the right, its speed is reduced by the wind and its speed with respect to the Earth is $c - v$. On its return journey, as the light beam moves to the left downwind, its speed with respect to the Earth is $c + v$. Thus, the time of travel to the right is $L/(c - v)$, and the time of travel to the left is $L/(c + v)$. The total time of travel for the round-trip along the horizontal path is

$$t_1 = \frac{L}{c + v} + \frac{L}{c - v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c} \left(1 - \frac{v^2}{c^2}\right)^{-1}$$

Now consider the light beam traveling perpendicular to the wind, as shown in Figure 1.4. Because the speed of the beam relative to the Earth is $(c^2 - v^2)^{1/2}$ in this case (see Fig. 1.3c), the time of travel for each half of this trip is $L/(c^2 - v^2)^{1/2}$, and the total time of travel for the round-trip is

$$t_2 = \frac{2L}{(c^2 - v^2)^{1/2}} = \frac{2L}{c} \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Thus, the time difference between the light beam traveling horizontally and the beam traveling vertically is

$$\Delta t = t_1 - t_2 = \frac{2L}{c} \left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

\(^5\)From an Earth observer's point of view, changes in the Earth's speed and direction in the course of a year are viewed as ether wind shifts. In fact, even if the speed of the Earth with respect to the ether were zero at some point in the Earth's orbit, six months later the speed of the Earth would be 60 km/s with respect to the ether, and one should find a clear fringe shift. None has ever been observed, however.
Because $v^2/c^2 \ll 1$, this expression can be simplified by using the following binomial expansion after dropping all terms higher than second order:

$$(1 - x)^n = 1 - nx \quad \text{(for } x \ll 1)$$

In our case, $x = v^2/c^2$, and we find

$$\Delta t = t_1 - t_2 = \frac{L v^2}{c^2} \quad (1.4)$$

The two light beams start out in phase and return to form an interference pattern. Let us assume that the interferometer is adjusted for parallel fringes and that a telescope is focused on one of these fringes. The time difference between the two light beams gives rise to a phase difference between the beams, producing the interference fringe pattern when they combine at the position of the telescope. A difference in the pattern (Fig. 1.6) should be detected by rotating the interferometer through 90° in a horizontal plane, such that the two beams exchange roles. This results in a net time difference of twice that given by Equation 1.4. The path difference corresponding to this time difference is

$$\Delta d = c(2\Delta t) = \frac{2L v^2}{c^2}$$

The corresponding fringe shift is equal to this path difference divided by the wavelength of light, $\lambda$, because a change in path of 1 wavelength corresponds to a shift of 1 fringe.

$$\text{Shift} = \frac{2L v^2}{\lambda^2} \quad (1.5)$$

In the experiments by Michelson and Morley, each light beam was reflected by mirrors many times to give an increased effective path length $L$ of about 11 m. Using this value, and taking $v$ to be equal to $3 \times 10^4$ m/s, the speed of the Earth about the Sun, gives a path difference of

$$\Delta d = \frac{2(11 \text{ m})(3 \times 10^4 \text{ m/s})^2}{(3 \times 10^8 \text{ m/s})^2} = 2.2 \times 10^{-7} \text{ m}$$

Figure 1.5 Albert A. Michelson (1852–1931). A German-American physicist, Michelson invented the interferometer and spent much of his life making accurate measurements of the speed of light. He was the first American to be awarded the Nobel prize (1907), which he received for his work in optics. His most famous experiment, conducted with Edward Morley in 1887, implied that it was impossible to measure the absolute velocity of the Earth with respect to the ether. Subsequent work by Einstein in his special theory of relativity eliminated the ether concept by assuming that the speed of light has the same value in all inertial reference frames.

(Niels Bohr Library, U.S.N.A./Courtesy of AIP Emilio Segre Visual Archive)

Figure 1.6 Interference fringe schematic showing (a) fringes before rotation and (b) expected fringe shift after a rotation of the interferometer by 90°.
This extra distance of travel should produce a noticeable shift in the fringe pattern. Specifically, using light of wavelength 500 nm, we find a fringe shift for rotation through 90° of

$$\text{Shift} = \frac{\Delta d}{\lambda} = \frac{2.2 \times 10^{-7}}{5.0 \times 10^{-7}} m = 0.40$$

The precision instrument designed by Michelson and Morley had the capability of detecting a shift in the fringe pattern as small as 0.01 fringe. However, they detected no shift in the fringe pattern. Since then, the experiment has been repeated many times by various scientists under various conditions, and no fringe shift has ever been detected. Thus, it was concluded that one cannot detect the motion of the Earth with respect to the ether.

Many efforts were made to explain the null results of the Michelson–Morley experiment and to save the ether concept and the Galilean addition law for the velocity of light. Because all these proposals have been shown to be wrong, we consider them no further here and turn instead to an auspicious proposal made by George F. Fitzgerald and Hendrik A. Lorentz. In the 1890s, Fitzgerald and Lorentz tried to explain the null results by making the following ad hoc assumption. They proposed that the length of an object moving at speed \( u \) would contract along the direction of travel by a factor of \( \sqrt{1 - v^2/c^2} \). The net result of this contraction would be a change in length of one of the arms of the interferometer such that no path difference would occur as the interferometer was rotated.

Never in the history of physics were such valiant efforts devoted to trying to explain the absence of an expected result as those directed at the Michelson–Morley experiment. The difficulties raised by this null result were tremendous, not only implying that light waves were a new kind of wave propagating without a medium but that the Galilean transformations were flawed for inertial frames moving at high relative speeds. The stage was set for Albert Einstein, who solved these problems in 1905 with his special theory of relativity.

1.4 POSTULATES OF SPECIAL RELATIVITY

In the previous section we noted the impossibility of measuring the speed of the ether with respect to the Earth and the failure of the Galilean velocity transformation in the case of light. In 1905, Albert Einstein (Fig. 1.7) proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time.\(^6\) Einstein based his special theory of relativity on two postulates.

1. **The Principle of Relativity:** All the laws of physics have the same form in all inertial reference frames.

2. **The Constancy of the Speed of Light:** The speed of light in vacuum has the same value, \( c = 3.00 \times 10^8 \text{ m/s} \), in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

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