Topics for Today:

- Announcements
  - Office hrs: 11-12pm M,W, 10-11am Fri
  - Office: EERC 614. Phone: 906.487.2857
  - XFMR exercises posted on web page, due Mon Sept 23rd 9am
  - CKTS problems collected by 9am Wed Sept 18th (latest).
  - Recommended problems from Ch.3 for review

- XFMR, Chapter 2 - Transformers and circuits w/transformers
  - Single phase transformers, basic structure
  - Winding R and Leakage, Core losses and saturation
  - 3-ph transformer banks and phase shifts (ANSI/IEEE vs. IEC)
  - Standard 30°shift transformers, non-standard connections
  - Pos/neg sequence phase shifts, sequence networks
  - Autotransformers
  - Load Tap Changing (LTC) transformers
REVIEW OF MAGNETIC CIRCUITS

As a simple example, an ideal single-winding magnetic circuit will be used. The magnetic core in this case is assumed to have no magnetic saturation, even at high levels of flux density.

Some of the basic parameters which physically define this circuit are:

A = Cross-sectional area of core
N = Number of turns of the winding
\( \lambda \) = Mean (average) path length of core (dashed line)
\( \mu \) = Magnetic permeability of the core. \( \mu \) depends on the type of core material. \((M=\mu_r\mu_0)\)

Some other important magnetic circuit quantities are defined as follows:

Reluctance of magnetic core:

\[ R = \frac{\lambda}{\mu A} \text{ H}^{-1} \]

Magnetomotive Force:

\[ \text{MMF} = N I \text{ Amp-Turns} \]

Magnetic Flux:

\[ \phi = \frac{\text{MMF}}{R} = \frac{N I}{R} \text{ Webers} \] (Direction given by "right-hand rule").

Magnetic Field Intensity:

\[ H = \frac{\text{MMF}}{\lambda} \frac{\text{Amp-Turns}}{m} \]

Magnetic Flux Density

\[ B = \frac{\phi}{A} \text{ Webers/m}^2 \text{ or Tesla} \]
Flux Linkage

\[ \lambda = N \phi = N A B \text{ Weber-Turns (or Volt-sec)} \]

Inductance

\[ L = \frac{\lambda}{I} = \frac{N^2 A}{R} = \frac{N^2 M A}{\lambda} \text{ Wb-Turns Amp} \text{ or Henries} \]

Induced Voltage

\[ v(t) = \frac{d\lambda}{dt} = N \frac{d\phi}{dt} = L \frac{dI}{dt} \text{ Volts} \]

Note that in this case, the induced voltage \( v(t) \) is zero, since the current and flux do not change with time.

USE OF VARIOUS UNITS OF MEASUREMENT

Manufacturer's test reports for various magnetic materials may give parameters in several different units of measurement. The following is a clarification of these different units:

Flux Density (B)

Standard Unit: Tesla = Weber/m^2
Other Unit: Maxwells (lines/inch^2) = Tesla \times 64500
Other Unit: Gauss = Tesla \times 10^4

Field Intensity (H)

Standard Unit: Ampere-Turns/m or Amps/m or Amps/cm
Other Unit: Oersteds = Ampere-Turns/m \times 0.01257

* Note that "Turns" is not really a dimensional unit

Magnetomotive Force (MMF)

Standard Unit: Ampere-Turns
Other Unit: Gilberts = Ampere Turns \times 0.4\pi
\[ R = \frac{\mu_0}{nA} \quad \text{Magnetic} \quad R = \frac{1}{\sigma A} \quad \text{Electrical} \]

\[ \Phi R = NI \quad \text{Amperes Law} \]

\[ \vec{H} = \frac{\text{MMF}}{l} \quad \text{A/m} \quad \vec{E} = \frac{\text{EMF}}{l} \quad \text{V/m} \]

\[ B = \frac{\Phi}{A} \quad \text{wb/m}^2 \quad J = \frac{\vec{I}}{A} \quad \text{A/m}^2 \]

\[ B = \mu_0 m. H \]
SATURABLE MAGNETIC CIRCUITS

As an example, we consider a 1Ω two-winding transformer with a saturable magnetic core.

If flux leakage, winding resistance, and core losses are included, the following equivalent circuit can be used:

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Note: \( \tilde{V}_p \) & \( \tilde{V}_s \) are voltages measured at terminals. 
\( \tilde{E}_p \) & \( \tilde{E}_s \) are magnetically induced via core.

- \( R_1 \) and \( R_2 \) : AC resistance of windings (linear)
- \( R_c \) : Resistance representing core losses (nonlinear)
- \( L_m \) : Magnetizing inductance of the core (nonlinear)
- \( L_{L1} \) & \( L_{L2} \) : Leakage inductance of windings (linear)

Note that core losses consist of eddy current losses and hysteresis losses, which can be frequency dependent and voltage dependent. Therefore, \( R_c \) cannot be represented as a linear resistance.
Note also that the magnetizing inductance $L_m$ is nonlinear due to magnetic saturation and cannot be represented as a linear inductance.

In other words, $\mu$ is not constant, and $B$ & $H$ are not linearly related. Therefore, behavior of the transformer core depends on the $B$-$H$ relationship at each instant of time. This nonlinearity must be included when doing transient analysis of transformers.

Hysteresis makes the behavior of the core more complicated than the above $B$-$H$ characteristic indicates. The areas of the hysteresis loops shown below (left) are proportional to the energy required for one steady-state cycle of operation. Each loop corresponds to a different level of voltage. If operation is not steady-state (below, right), a subloop or "re-entrant loop" is followed.
Leakage inductance arises because not all the flux links the windings via the core. Figure 4-1 shows an example in which some flux has leaked from the iron core and completed the magnetic circuit through the air. Such leakage is associated with both the primary and secondary windings. For convenience of illustration a core-type transformer with windings on separate limbs is shown. The principle, however, applies to any transformer (or inductor for that matter). If the windings are placed one on top of the other, as is more usual, there will still be leakage inductance, but probably to a lesser degree.

The effect of leakage inductance is as though a small part of the total inductance had been detached and placed in series with the winding, as shown schematically in Fig. 4-2, where \( L_p' \) and \( L_s' \) are the primary winding and secondary winding leakage inductances, respectively. Again, the effect is generally not important except at relatively high frequencies, for then the reactances are high, and being in series, have a marked effect on performance.
Three-Phase Transformers

All of these can and are used to indicate the same winding connections:

IEEE Stds:

Schematic:

Circuit 3-line diagram:

In Europe and much of the world:

IEC Stds: Dyn11

"Old-down" diagram:
How many possibilities are there for Δ-Y or Y-Δ phase shifts?

\[ \begin{align*}
\pm 30^\circ \\
\pm 90^\circ \\
\pm 150^\circ \\
\end{align*} \right) \quad 6 \text{ each}
\implies 12 \text{ total.}

Auto-Δ
Zig-Zig
Extended Δ.
\[ V_{A1} = V_{01} \left(1 / 30^{\circ}\right) \]

PRI POS SEQ VOLTAGES

SEC POS SEQ VOLTAGES

PRI POS SEQ CURRENTS

SEC POS SEQ CURRENTS

\[ V_{A2} = V_{02} \left(1 / -30^{\circ}\right) \]

PRI NEG SEQ VOLTAGES

SEC NEG SEQ VOLTAGES

PRI NEG SEQ CURRENTS

SEC NEG SEQ CURRENTS

ANSI STANDARD 30-DEGREE SHIFT WYE-DELTA
ANSI STANDARD 30-DEGREE SHIFT DELTA-WYE