$$\tilde{S}_{1} = V_2 I_{1a}^* = (.963 \angle -5.\cancel{0}^\circ)(.4732 \angle 25.536) = .427 + j.16$$

~~tempose~~

$$\tilde{S}_2 = V_2 \frac{I_{1b}^*}{a} = .963 \angle -5.1^\circ \frac{(.59 \angle -46.75)^*}{1.05} = .406 + j.3616$$

$$\tilde{S} = V_2 I_2^* = (.963 \angle 5.1)\left(\frac{V_2}{Z}\right)^* = .983 \angle 32^\circ = \tilde{S}_1 + \tilde{S}_2$$

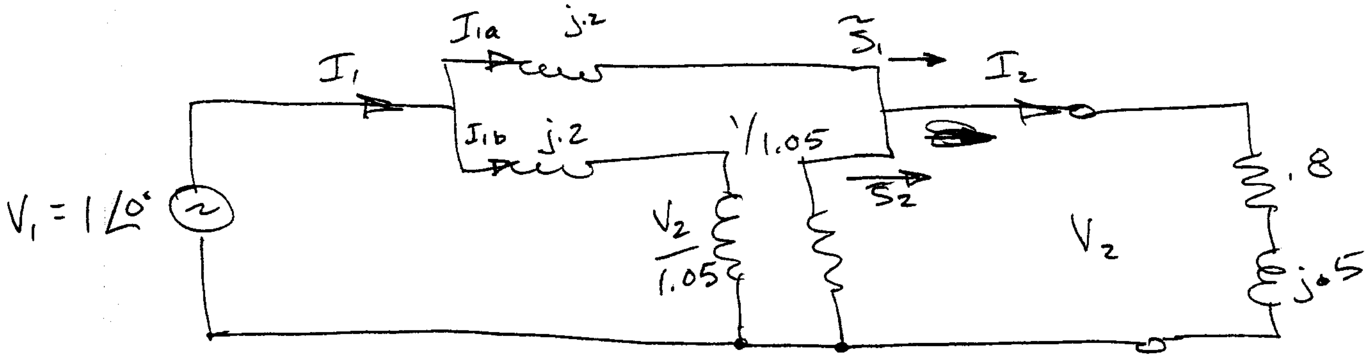
	Before TC	After T.C. Circ. Approx.	After T.C. Ckt. Analysis
V_1	1	1	1
V_2	$.943 \angle -4.86^\circ$	$.963 \angle -4.87^\circ$	$.963 \angle -5.1$
at V_2: P_1	.4	.398	.427
at V_2: P_2	.4	.418	.406
at V_2: Q_1	.25	.135	.159
at V_2: Q_2	.25	.375	.362

~~So Tap Changing XFMR shifted Q to XFMR with tap turn to higher than rated secondary voltage.~~ ~~higher tap setting.~~

So:

Q is shifted to XFMR of higher tap setting.
P is still divided almost evenly.

Now add tap changer $a = 1.05$



$$I_1 = I_{1a} + I_{1b}$$

$$I_1 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}$$

$$I_2 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}\left(\frac{1}{1.05}\right)$$

$$V_2 = I_2(.8 + j.5) = \left[\frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}\left(\frac{1}{1.05}\right)\right](.8 + j.5)$$

$$V_2 = \left(1 - V_2 + \frac{1 - \frac{V_2}{1.05}}{1.05}\right)\left(\frac{.8 + j.5}{j.2}\right) = \left(1.952 - 1.907 V_2\right)\left(4.717\angle -58°\right)$$

$$V_2 = -1.907 V_2 (4.717\angle -58°) + (1.952)(4.717\angle -58°)$$

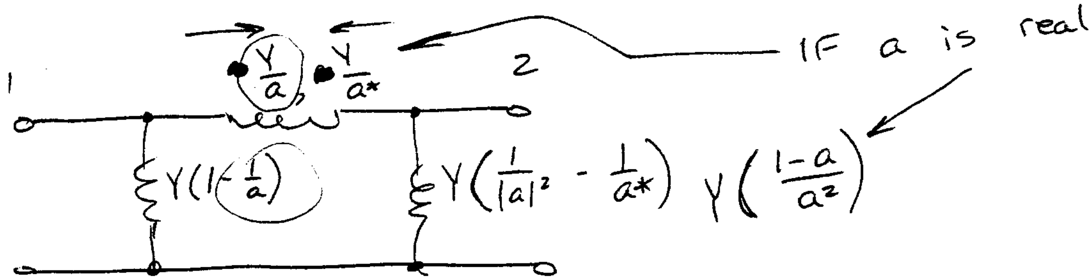
$$= -8.995\angle -58° \, V_2 + 9.207\angle -58°$$

$$V_2 = \frac{9.207\angle -58°}{1 + 8.995\angle -58°} = \frac{9.207\angle -58°}{9.563\angle -52.9} = .963\angle -5.1°$$

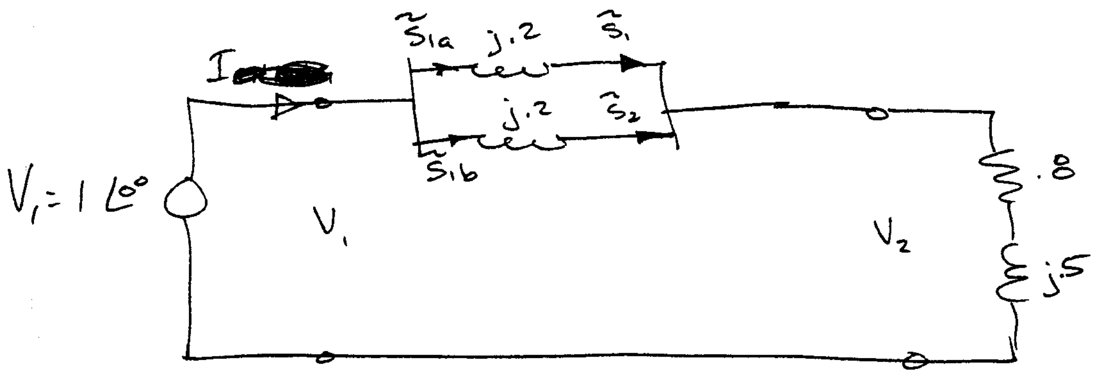
$$I_{1a} = \frac{1 - V_2}{j.2} = .4732\angle -25.54° \qquad I_{1b} = \frac{1 - \frac{V_2}{1.05}}{j.2} = .59\angle -46.75°$$

$$S_{1a} = V_1 I_{1a}^* = .427 + j.204$$

$$S_{1b} = V_1 I_{1b}^* = .4067 + j.4324$$



2) CIRCUIT THEORY APPROACH



$$V_2 = \frac{(.8+j.5)(1\angle 0°)}{.8+j.6} = .94-j.08 = .9434\angle -4.86°$$

$$I_2 = \frac{.94-j.08}{.8+j.5} = 1\angle -36.87°$$

$$\tilde{S} = VI^* = .9434\angle -4.86°\,(1\angle 36.87°) = .9434\angle 32° = .8+j.5$$

$$\left.\begin{aligned} S_1 &= .4+j.25 \\ S_2 &= .4+j.25 \end{aligned}\right\} = \frac{S_{TOTAL}}{2}$$

$$I_1 = \frac{1}{.8+j.6} = 1(.8-j.6) = .8-j.6$$

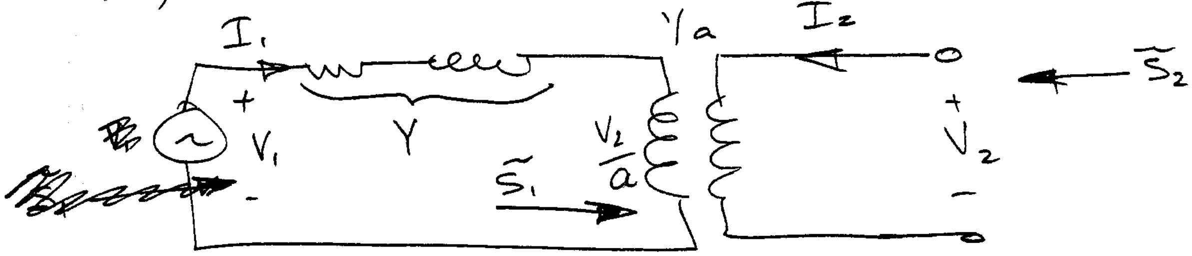
$$S_{1a} = \frac{V_1 I_1^*}{2} = .4+j.3$$

$$S_{1b} = \frac{V_1 I_1^*}{2} = .4+j.3$$

Difference due to XFMR Inductance

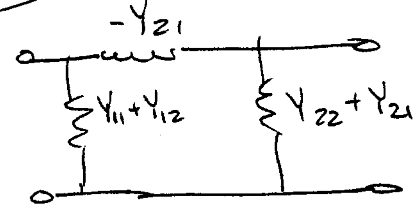
METHOD 1

So,



So we must find a way to model Y for Y_{12} as $1/a$ varies.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{12}V_1 + Y_{22}V_2$$



IN GENERAL

$$\tilde{S}_1 = -\tilde{S}_2$$

$$\tilde{S}_1 = \frac{V_2}{a} I_1^* \qquad S_2 = V_2 I_2^*$$

$$\frac{V_2}{a} I_1^* = -V_2 I_2^*$$

$$I_1^* = -a I_2^*$$

$$I_1 = -a^* I_2$$

$$I_1 = \left(V_1 - \frac{V_2}{a}\right)Y = YV_1 - \frac{Y}{a}V_2 = -a^* I_2$$

$\therefore$

$$I_2 = \frac{-I_1}{a^*} = \frac{-YV_1}{a^*} + \frac{Y}{aa^*}V_2$$

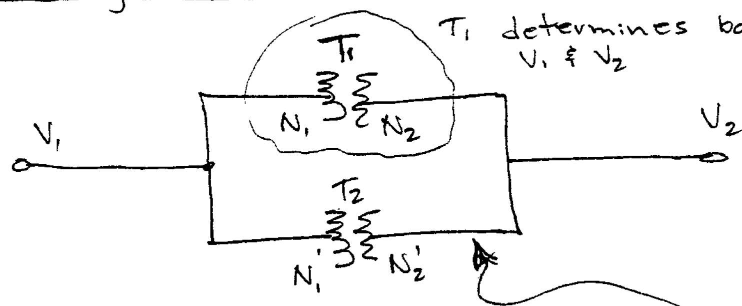
$\therefore$

$$Y_{11} = Y \qquad Y_{12} = -\frac{Y}{a}$$

$$Y_{21} = -\frac{Y}{a^*} \qquad Y_{22} = \frac{Y}{aa^*} = \frac{Y}{|a|^2}$$

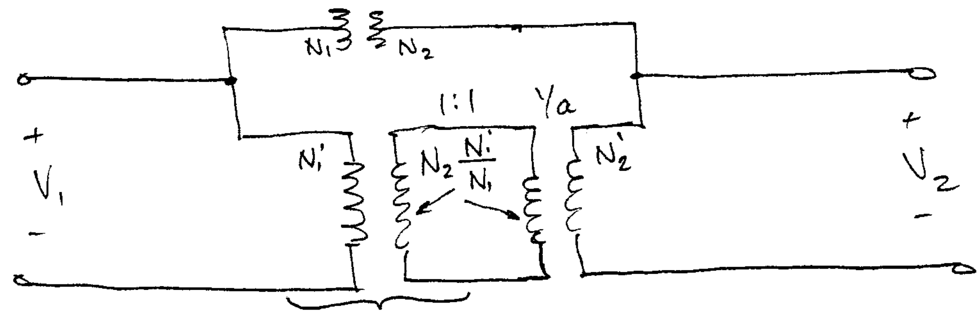
EE5200 - Study Prior to 9/28 Lecture

Paralleling Transformers of Unlike Turns Ratio



$$\frac{N_1}{N_2} \cong \frac{V_1}{V_2}$$

What happens for $\frac{N_1'}{N_2'} \neq \frac{N_1}{N_2}$?

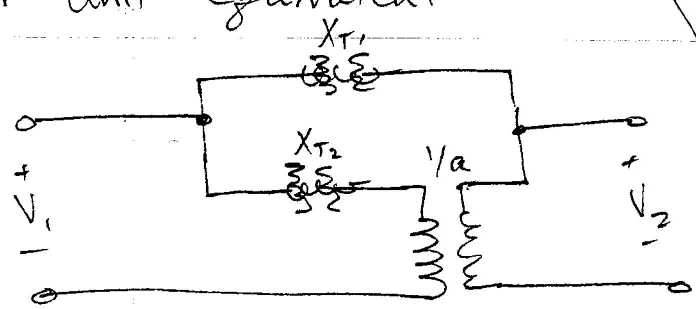


Replace T_2 with 2 XFMRS: - First is same ratio as T_1 $\frac{N_1}{N_2} = \frac{N_1'}{X}$

$$X = N_2 \frac{N_1'}{N_1}$$

Second XFMR has ratio of off-nominal turns

Per unit equivalent:



$$\frac{1}{a} = \frac{\left(N_2 \frac{N_1'}{N_1}\right)}{N_2'} = \frac{N_2 N_1'}{N_1 N_2'}$$

$$\therefore \boxed{a = \frac{N_1}{N_2} \frac{N_2'}{N_1'}} = \text{p.u. turns ratio}$$

Three Methods to Analyze:
1) Admittance Method
2) Circuit Theory
3) Circulating Current Method (Approximate)

$$I_a = \frac{1.031 + j.013 - 1}{j.1} = .131 - j.311$$

$$\frac{I_b}{a^*} = I_2 - I_a = .8 - j.6 - .131 + j.311 = .669 - j.289$$

$$S_a = .131 + j.311$$
$$S_b = .669 + j.289$$

NOTE SHIFT IN POWER FLOW THRU XFMRS

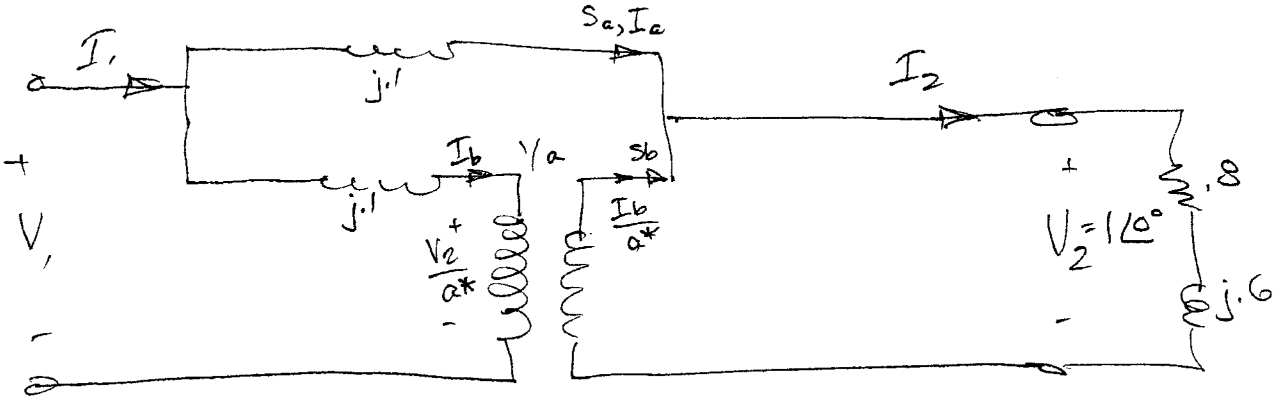
	BEFORE	AFTER
I_a	$.4 + j.3$	$.131 - j.311$
I_b/a^*	$.4 - j.3$	$.669 - j.289$
P_a	$.4$	$.131$
P_b	$.4$	$.669$
Q_a	$+.3$	$.311$
Q_b	$+.3$	$.289$
V_1	$1.031 \angle 2.2^\circ$	$1.031 \angle .73^\circ$
V_2	$1.0 \angle 0^\circ$	$1.0 \angle 0^\circ$

if a is at positive angle, power flow through phase shifting XFMR increases

$$S_{1a} = V_2 I_{1a}^* = \left(.963 \angle -4.87^\circ\right)\left(.437 \angle -23.63\right)^* = .421 \angle 18.76^\circ$$
$$= \boxed{.398 + j.135}$$

$$S_{2a} = V_2 I_{2a}^* = \left(.963 \angle -4.87^\circ\right)\left(.584 \angle 46.73\right) = .562 \angle 41.86$$
$$= \boxed{.418 + j.375}$$

Phase ~~Changing~~ Shifting XFMR



$$a = 1 \angle 3^\circ = e^{j\pi/60}$$

Without phase shifter,

$$P_{LOAD} = \frac{V_2^2}{Z^*} = 1(.8 + j.6) = .8 + j.6$$

$$S_a = .4 + j.3$$
$$S_b = .4 + j.3$$
$$I_a = .4 - j.3$$
$$I_b = .4 - j.3$$

$$V_1 = 1\angle 0^\circ + (.4 + j.3)(j.1)$$
$$= 1.0307 \angle 2.2^\circ$$

With Phase Shifter,

$$\frac{V_1 - 1}{j.1} + \frac{V_1 - \frac{1}{1\angle -3^\circ}}{j.1}\underbrace{\left(\frac{1}{1\angle -3^\circ}\right)}_{\frac{1}{a^*}} = \frac{1}{.8 + j.6} = I_2$$

$$V_1\left(1 + \frac{1}{1\angle -3^\circ}\right) - 1 - \left(\frac{1}{1\angle -3^\circ}\right)^2 = j.1/(.8 + j.6)$$

$$V_1\left(1.999 \angle -1.5^\circ\right) \cancel{- 1 - (.9945 - j.1045) = .1\angle 53.13 + 1.9945 - j.1045}$$

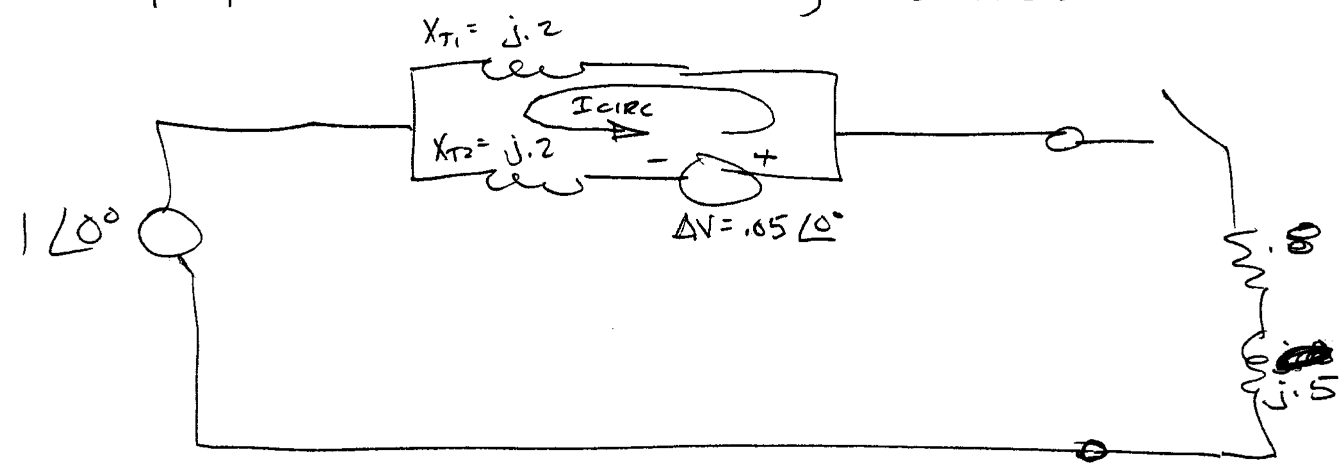
$$\cancel{V_1 = 2.05 \angle .8^\circ / 1.999 \angle -1.5^\circ = 1.028 \angle .82 = 1.028 + j.016}$$

$$V_1 = 1.031 \angle .73^\circ = 1.031 + j.013 \Rightarrow$$

$$\cancel{I_a = \frac{1.028 + j.016 - 1}{j.1} = .322 \angle -60.3^\circ = .159 - j.28}$$

Approximate solution:

Superposition of circulating current



First, look at XFMR before load applied.

$$I_{CIRC} = \frac{.05 \angle 0^\circ}{j.4} = .125 \angle -90^\circ = -j.125$$

From first example,

$$I_{1a} = .4 - j.3 + \underbrace{j.125}_{I_{CIRC}} = .4 - j.175$$

$$I_{1b} = .4 - j.3 - j.125 = .4 - j.425$$

$$\cancel{V_2 = V_1 - (I_{1a})(j.2) = 1 - (.4 - j.125)(j.2) = 1 - (j.08 + .025) = .98 \angle -4.45}$$

By superposition,

$$V_2 = V_{2\ \text{BEFORE TAP CHANGE}} + \frac{\Delta V (jX_{T_1})}{jX_{T_1} + jX_{T_2}} \left[\frac{Z_{LOAD}}{Z_{TOTAL}} \right]$$

Voltage at Load due to ΔV

$$\Delta V_{oc} = \frac{\Delta V (jX_{T_1})}{(jX_{T_1} + jX_{T2})}$$

$$Z_{th} = jX_{T_1} \| jX_{T2}$$

$$= .943 \angle -4.86^\circ + \underbrace{\frac{.05(j.2)}{j.4}}_{\Delta V_{oc}} \left[\frac{.8 + j.5}{\underbrace{.8 + j.6}_{j.5 + (jX_{T_1} \| jX_{T_2})}} \right]$$

$$V_2 = .963 \angle -4.87^\circ$$