Topics for Today:

• Announcements
  • Detailed term project outlines (i.e. Table of Contents + List of references
  • Software: you can apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  • ASPEN software - remote.mtu.edu
  • Office hrs: EERC 123, WF 4-6pm. Instructor’s office: EERC 614
  • Recommended problems & all solutions: Ch.7 solns posted.

• Chapter 7 - Network Equations, Admittance Approaches
  • How’s your linear algebra? Time to make use of it...
  • Basic strategy for building up [Y] for whole network
  • Quick recap of xfmrs and lines.
  • Generators
  • Example of building [Y] for 4-bus system.
  • Network Reduction (Kron Reduction)
  • Solution of matrix equations (system of linear equations)
  • Upcoming homework - intro to Matlab, matrices, equations.
- Spreadsheet
- ASPEN
  - LF
  - SC
- phaser
- Arc Flash
- Relay Coordination
- CYME - Optimal LF
- ATP
  - Line Constants
  - Time-domain
- Feedback control
  - Relay Prot
- Matlab - Simulink
  - Sim Power
Close look at $z = a + j\beta$

$$y = \sqrt{2y} = \sqrt{(1.2 + j0.6) 0.2 \text{ mi}} = \sqrt{1.713 \text{ m/s/mi}}$$

$$= \frac{0.0033 \text{ m/s/mi}}{0.00212 \text{ rad/mi}}$$

$$= \frac{0.0034 \text{ nper/mi}}{0.00212 \text{ rad/mi}}$$

Tell us how much attenuation/mi

For 2.56 mi:

$$\text{Att} = 0.00034 \text{ per mi} (2.56 \text{ mi})$$

$$= 0.0085 \text{ or 8.5%}$$

Very high, yet assume more lossless?
\[ Z_c = \sqrt{\frac{Z}{3}} \implies \text{Real for lossless.} \]
\[ \approx \frac{300 \Omega}{(250-400 \Omega)} \approx 70-80 \Omega \text{ (cables)} \]

\[ Z_c = \sqrt{\frac{R+jXL}{jBC}} \]

\[ \rightarrow \]

\[ \uparrow \]
Reflections

Voltage Reflection Coefficient:
\[ \frac{V_R^-}{V_R^+} = \frac{Z_a - Z_c}{Z_a + Z_c} = P_R \]
\[ \frac{Z_s - Z_c}{Z_s + Z_c} = P_s \]

Current Reflection Coefficient
\[ \frac{i_R^-}{i_R^+} = -\frac{V_R^-}{V_R^+} = -P_R \]
The diagram shows an electrical circuit with a switch labeled "close switch." The time constant $T = 0.016\,s$ is noted, and a waveform indicating the current $I = \text{?}$ is also depicted.
\[
\begin{bmatrix} Z_B \end{bmatrix} = \begin{bmatrix} Y_B \end{bmatrix}^{-1}
\]

\[
\Rightarrow
\]

treat as off-nominal turns ratio.

\[
\text{still only need modify}
\]

\[
\text{y55} \quad \text{y57} \quad \text{y75} \quad \text{y77}
\]
Phase Shift XFMRS (Fig. 2.22)

Read §2.9!
→ \[ Y_{Bus} \]

Building by inspection:

\[
\begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{3} & -\frac{1}{4} \\
-\frac{1}{2} & 1 & -\frac{1}{3} & -\frac{1}{4} \\
-\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{4} \\
-\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_{23} & y_{24} \\
y_{33} & y_{34} \\
y_{43} & y_{44} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\frac{1}{3} - y_{34} & -y_{34} \\
-\frac{1}{4} - y_{44} & -y_{44} \\
\end{bmatrix}
\]

From KCL:

\[
\sum I_s \text{ in } = 0
\]
\[
y_{33} = y_{33} + y_{3-4} \\
y_{44} = y_{44} + y_{4-3} \\
y_{34} = y_{34} - y_{3+4} \\
y_{43} = y_{43} - y_{4+3}
\]

\[
y_{3-4} = y_{4-3} \text{ if bilateral.}
\]

**FACTS -**
- Non-bilateral: \(y_{mn} \neq y_{nm}\)

**EX:**
- UPFC - \(P+Q\)
- SVC - \(S\) and \(Q\)
- P.S. Transformer
Four Cases

Next:

\[
\begin{bmatrix}
  y_0 & y_{12} \\
  y_0 & y_{12}
\end{bmatrix}
\]
Basis Approach: Develop $\pi$-Equiv and handle just like T-Line.

One-Line:

\[
\begin{align*}
&1 \quad \frac{a:1}{35} \quad 2 \\
\end{align*}
\]

per-unit

per-phase

\[
\begin{align*}
&1 \quad \frac{a:2}{\cos}\quad \frac{\text{REF}}{\text{REF}}
\end{align*}
\]

Top-Changers

- LTC's
- Phase-Shift

\[
\begin{align*}
&\text{Nominal} \quad \text{Turns} \quad \text{Ratio} \\
&\pm \text{Adjustment} \quad \text{in phase angle} \quad (PS) \quad \text{or Volt mag} \quad (LTC)
\end{align*}
\]
Tap Changing XFMRS - Variations (p.u. representations)

\[ y_{sc} = \frac{1}{R + jX} \]

1. \( C \) is off-nominal turns ratio. In general, \( C \) is complex.
2. \( C \) is real for LTC.
3. \( C \) is complex for PS.
4. If \( |C| \neq 1 \) then magnitude change.
5. If \( C \) is complex, phase shift.

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TAP-CHANGERS

On One-Line Diags:

Conceptually:

In per unit, nominal transformation "disappears"
Generically, we can describe this as a 2-node \([Y]\) as

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix}
\]

where

\[
\frac{V_1}{V_2} = \frac{I_1}{I_2}
\]

\[
\begin{bmatrix}
\frac{V_1}{V_2}
\end{bmatrix}
= \begin{bmatrix}
\frac{I_1}{I_2}
\end{bmatrix}
\]
**Standard Approach:**

\[
\begin{bmatrix}
 y_{11} & y_{12} \\
 y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
 V_1 \\
 V_2
\end{bmatrix}
= \begin{bmatrix}
 I_1 \\
 I_2
\end{bmatrix}
\]

**Goal:**

\[
y_{11} = y_{SER} + y_{SH1} \\
y_{12} = -y_{SER} \\
y_{21} = -y_{SER} \\
y_{22} = y_{SER} + y_{SH2}
\]
Strategically using shorts, we can isolate on the values of \([Y]\).

\[
y_{11} = \frac{\bar{I}_1}{\bar{V}_1}, \quad \bar{V}_2 = 0
\]

\[
= \frac{1}{Z_{\text{EQ}}} = Y_{\text{EQ}} = \frac{-\bar{I}_2}{\bar{V}_2}, \quad \bar{V}_1 = 0
\]

\[
= \frac{1}{Z_{\text{EQ}}/|C_1|^2} = |C_1|^2 Y_{\text{EQ}}
\]
\[ \vec{I}_1 = -\frac{c \vec{V}_2}{2 \varepsilon_0} \quad \vec{I}_2 = -\vec{I}_1 \times \mathbf{c}^* = - \left[ \frac{c \vec{V}_2}{2 \varepsilon_0} \right] \mathbf{c}^* \]

**Note:** \[ \frac{\vec{I}_2}{\vec{I}_1} = \mathbf{c}^* \]

\[ \vec{I}_2 = \frac{|c| \vec{V}_2}{2 \varepsilon_0} \]
\[ y_{12} = \left. \frac{\tilde{I}_1}{\tilde{V}_2} \right|_{\tilde{V}_1 = 0} = -\frac{c}{\tilde{V}_2/\tilde{Z}_{eq}} = -cY_{eq} \]

\[ y_{21} = \left. -\frac{\tilde{I}_2}{\tilde{V}_1} \right|_{\tilde{V}_2 = 0} = -\frac{c*\tilde{I}_2}{\tilde{V}_1} = -c*Y_{eq} \]

\[ S_{in} = \tilde{V}_1 \tilde{I}_1^* = \tilde{V}_2 \tilde{I}_2^* = S_{out} \]

Note: Ideal XFRH, by definition, has "C" is Voltage ratio. 
\[ C = \frac{\tilde{V}_1}{\tilde{V}_2} \Rightarrow \frac{\tilde{I}_2^*}{\tilde{I}_1^*} = c \]
If we "reverse engineer" our network, then \([Y]\) into an equivalent 2-bus network, then:

\[
\begin{align*}
I_1 &= C_{\text{YEQ}} V - Y_{\text{EQ}} (1 - c) \cdot V \\
V_1 &= Y_{\text{EQ}} (1 - c) \cdot V
\end{align*}
\]
Observations:

- LTC (TCUL) has a c that is real.
  \[ \text{Transfer Admittances} \]
  \[ C \cdot Y_{\text{eq}} = C \cdot Y_{\text{eq}} \]
  \[ \Rightarrow \text{Bilateral}, \quad (y_{12} = y_{21}) \]

- Phase-Shifter (PS) has complex c.
  \[ \text{Transfer admittances} \]
  \[ C \cdot Y_{\text{eq}} \neq C \cdot Y_{\text{eq}} \]
  \[ y_{12} \neq y_{21} \]
  \[ \text{Not Bilateral.} \quad \text{[Y] not symm. about main diag.} \]
Transformer LTC's in the CDF File Format

Tap and impedance location specified in first two entries in branch data section.
- entry 1 is bus non-unity tap is connected to
- entry 2 is bus device impedance is connected to

Complex turns ratio due to phase shifting transformer split to two entries
- entry 15 is transformer final turns ratio
- entry 16 is transformer (phase shifter) final angle

Examples:

Example 1:

<table>
<thead>
<tr>
<th>Entry:</th>
<th>1</th>
<th>2</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>7</td>
<td>.975</td>
<td>0</td>
</tr>
</tbody>
</table>

Example 2:

<table>
<thead>
<tr>
<th>Entry:</th>
<th>1</th>
<th>2</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>30</td>
</tr>
</tbody>
</table>
Ckt 1

Ckt 2, inductive coupling!

R1, jX1

R2, jX2

B = 5/MI
Mutual Inductance

\[ \vec{E}_1 \rightarrow \vec{A} \rightarrow \vec{E}_2 \rightarrow \vec{B} \rightarrow \vec{E}_1' \]

\[ \vec{H}_1 \rightarrow \vec{B} \rightarrow \vec{H}_2 \rightarrow \vec{A} \rightarrow \vec{H}_1' \]

END VIEW
MUTUAL INDUCTANCE

See also handout on Basic Magnetic Circuits

Fundamental definition of inductance: \( L = \frac{\Phi}{i} = \frac{N\Phi}{i} \)

Self-Inductance

\( L_{11} = \frac{N_1 \phi_{11}}{i_1} = \frac{\phi_{11}}{i_1} = \frac{N_1^2}{R} \)

Mutual Inductance

\( L_{12} = \frac{N_1 \phi_{12}}{i_2} = \frac{\phi_{12}}{i_2} = \frac{N_1 N_2}{R} \)

\( L_{21} = \frac{N_2 \phi_{21}}{i_1} = \frac{\phi_{21}}{i_1} = \frac{N_2 N_1}{R} \)

\( L_{22} = \frac{N_2 \phi_{22}}{i_2} = \frac{\phi_{22}}{i_2} = \frac{N_2^2}{R} \)
How to Use the Concept of Mutual Inductance

Two-Port Device:

\[
\begin{bmatrix}
    i_1 \\
    i_2 \\
\end{bmatrix}
= \begin{bmatrix}
    L_{11} & L_{12} \\
    L_{21} & L_{22} \\
\end{bmatrix}
\begin{bmatrix}
    \frac{di_1}{dt} \\
    \frac{di_2}{dt} \\
\end{bmatrix}
\]

Note: Reference direction of currents is into terminals at (+) side of voltage.

In time domain:

\[
\begin{bmatrix}
    v_1 \\
    v_2 \\
\end{bmatrix}
= \begin{bmatrix}
    L_{11} & L_{12} \\
    L_{21} & L_{22} \\
\end{bmatrix}
\begin{bmatrix}
    \frac{di_1}{dt} \\
    \frac{di_2}{dt} \\
\end{bmatrix}
\]

In phasor domain:

\[
\begin{bmatrix}
    v_1^{(\omega)} \\
    v_2^{(\omega)} \\
\end{bmatrix}
= \begin{bmatrix}
    j\omega L_{11} & j\omega L_{12} \\
    j\omega L_{21} & j\omega L_{22} \\
\end{bmatrix}
\begin{bmatrix}
    i_1^{(\omega)} \\
    i_2^{(\omega)} \\
\end{bmatrix}
\]

Also of note:
In some texts, since \(L_{12}\) and \(L_{21}\) are mutual inductances, they are called \(M_{12}\) and \(M_{21}\). Same thing.
$[\mathbf{Z}'] = [\mathbf{Y}]$

\[
\begin{bmatrix}
y_1 & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
y_1' \\
y_2'
\end{bmatrix}
= 
\begin{bmatrix}
y_1' \\
y_2'
\end{bmatrix}
\]
Assume high $z/R$ ($R \to 0$)

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix} Z & I_1 \\
& I_2
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} \Rightarrow \begin{bmatrix} Y_1 & V_1 \\
& V_2
\end{bmatrix} = \begin{bmatrix} I_1 \\
I_2
\end{bmatrix}
\]

pre-multiply both sides by $\begin{bmatrix} Z \end{bmatrix}^{-1}$